TOPICS, TERMS and FORMULAS, ASTR 502 (Fall 2017)

Here is the set of topics and terms for the first quiz. I PROMISE the next quiz will have much less math!   If you don't have a calculator, you can get sines and cosines, and logarithms from Google calculator – give it a try! (but use, ***and bring,*** a real calculator for the quiz)
<mark>**HARVEY UPDATE:** Material in yellow highlighter will not be tested this year!!</mark>

Here are text versions of formulas, if you don't have super and subscripts…
a * b   means a times b (the product of two terms multiplied together).   Don't use "x" which can be a variable.
a / b   means a divided by b
a**b   or a ^ b   means a raised to the b power: a^b; a**2 is a-squared, a**3 is a-cubed, etc.
aEb   means a times ten to the b power. Thus 4.3E4 is 43,000 or 4.3×10^4
    Note: easy scientific notation - also shows the number of significant figures at the same time!
    Significant figure warning: never give an answer that has more digits than the most in your source data, and in fact it's better to give only the number of digits in your worst known parameter! So, for example, if you know the period to 3 significant digits, your calculator might give you an answer for the distance with 9 digits, but only 3 are significant, so only put three down! I will take off for answers with too many digits!
Sqrt (a) means the square root of a. Same as a**.5 $= \sqrt{a}$
Sin (α), Cos (α), Tan (α) means sine, cosine, and tangent of the Angle α.
    IF NO UNITS, it's measured in RADIANS (see below).
theta $= \theta =$ Greek theta (usually a measure of angle)
lambda $= \lambda$ = Greek lambda (latitude, or sometimes wavelength)
alpha $= \alpha =$ Greek alpha   (usually a measure of angle)
$\sim$ or $\approx$ or $\cong$ means "nearly/approximately/almost equal to"   (the last is the closest to exactly equal)
$\propto$ means "proportional to", so if a $\propto$ b then there exists a scalar k such that a = kb for all a and b.
$\equiv$ means "equal by definition"

deg = degrees (usual units, 360 deg $= 360° =$ a circle)
pi $= \pi =$ you know this one. Approximately 3.14159…. my favorite number $\cong \sqrt{10}$
R = the radius of a circle
Re = the radius of the Earth (generally written with as the "e" is an UPPERCASE subscript R_E).
qv = means "quid videre" which means "go look it up" (it's defined elsewhere).
v = velocity (vector);   V = Volume
G = gravitation constant $= 6.67$ E-11 (N-m**2/kg**2)

1. **latitude:** angle measured from the equator towards the pole. Pole is +90 deg. South pole is -90 degrees. ***Know that Houston is near 30 deg.*** Often shown as Greek lambda. (λ).

2. **longitude**: angle measured around the equator, positive towards the East. Zero at the Prime Meridian (Greenwich) (zero is arbitrary but "England ruled the waves"). ***Know that Houston is at approximately -95 degrees longitude.***   Often shown as Greek phi. (ϕ)

3. **Circumference of a circle:** the distance around a circle's edge. Equals $2 \pi * R$ where R is the radius of the circle.

4. **Circumference of the Earth**: almost exactly 40,000 km. Approximately 24,900 miles.

One mile ~ 1609 meters so you can always multiply by "1" (1.6 km / 1 mi) to change units. ***ALWAYS ALWAYS SHOW UNITS***!!!

5. **Radius of the Earth**: 6378 km. Approximately 4000 miles. I often use this as R_E.

6. **Corotation speed** at the equator: = circumference/one day = 24,906 miles / 24 hours = 1038 MPH or 1670 km/h. (at other latitudes it is 1038 cos (λ) MPH.

7. **Right ascension**: stellar coordinate similar to longitude, but generally measured in HOURS. (1 hour = 15 degrees). Zero is defined to be the direction to the Sun at the vernal equinox.

8. **Sidereal Time**: Right ascension of the stars on your meridian (line going from north star to southern horizon).   Since zero Right Ascension is the direction of the SUN at the vernal equinox, then noon on Mar 22 is zero Sidereal Time, not twelve! What sidereal time is noon on Sept 22? Noon on Dec 21? ***Be able to draw a sketch of the Earth's orbit to estimate sidereal time for various times of day at various days of the year***.

9. **Declination**: stellar coordinate similar to latitude. Zero at equator; +90 deg at North Celestial Pole. Generally shown as a Greek "delta" (δ).   So the "celestial coordinates" of a star or planet are given as, e.g. RA 13:31, δ +41.33 (or as degrees, minutes, seconds: 15°31'45"). ***Be able to change between decimal degrees and degrees, minutes, seconds.***

10. **Keplers's Laws**: Know Kepler's laws and how to use them. Use ratios whenever possible and save calculations! ***Know that they were empirically found, not theoretically derived.***

  Kepler's first: the orbit of a planet around the Sun is an ellipse with the Sun at one focus.
  2nd: the line from the Sun to the planet in orbit sweeps out equal areas in equal times
  3rd: (harmonic law): the period of a planet T squared is proportional to the semimajor axis, a, cubed.   *(mnemonic – times square)*
      Special case for planets, comets, etc around the Sun: $(T \text{ (years)})^2 = (a \text{ (AU)})^3$.

11. **Syzygy**: locations of special orientations of the Earth, sun, and a Planet. Generally, the Earth-Sun-Planet (or Earth-Planet-sun) angle will be 0, 180, or 90 degrees. (see next two).

12. **Exterior (superior) Planet**: farther than Earth from the Sun. Syzygy points are: opposition (opposite direction from Sun), conjunction (same direction as Sun); Quadrature (Planet-Earth-Sun angle = 90 deg).

13. **Inferior Planet**: closer than Earth from the Sun. Syzygy points are: inferior conjunction (between Earth and Sun); superior conjunction (far side of Sun); Greatest Eastern and Greatest Western Elongation (maximum Planet-Sun-Earth angle). Note: when they are at greatest elongation, we will be at quadrature for them).

14. **scalar:** a measurement that has only a magnitude, not a direction (e.g. mass, density, speed).

15. **vector:** a measurement that has a magnitude AND a direction (e.g. velocity, gravity, electric force, magnetic force)

16. **Newton's first law** (law of inertia): An object at rest or traveling in a straight line at constant speed will continue unless acted upon by an external force.   (e.g. constant VELOCITY). A special case of Newton's second law.

17. **Newtons' second law** : $\underline{F} = m\,\underline{a}$ An external force leads to an acceleration = $\underline{F}/m$.
   where $\underline{a}$ is the change of the VECTOR velocity $\underline{v}$ with time (delta $\underline{v}$ / delta t).
   So a change in DIRECTION even at constant SPEED is an acceleration (go around a curve!)
    A constant Force along the velocity leads to a constantly-increasing velocity.
      $\underline{v}x\,(t) = \underline{v}_0 + \underline{a}\,t\ =\ \underline{v}_0 + \underline{F}/m\,t$ where $\underline{v}_0$ is the initial velocity
     So a falling object increases its velocity by 9.8 m/s, every second.   (until it reaches "terminal velocity", qv)
   A constant Force PERPENDICULAR to the initial velocity makes a direction change but not a speed change. Will make the object want to go in a circle (orbit). (see centripetal acceleration below)

18. **sines and cosines** of an angle: Consider a right triangle (one perpendicular angle). The sine of one of the acute angles is the length of the **opposite** leg divided by the hypotenuse. The cosine of one of the acute angles is the length of the **adjacent** leg divided by the hypotenuse. Since the two acute angles must add up to 90 degrees (since the sum of the angle of any triangles is 180 degrees), then

   sin (alpha) $\equiv$ cosine (90 - alpha) = opposite / hypotenuse
   cos (alpha) $\equiv$ sin (90 – alpha) = adjacent / hypotenuse
   tan (alpha) $\equiv$ sin (alpha) / cos (alpha) = opp / adj
   sin**2(alpha) + cos**2(alpha) = 1   (true for any angle alpha; from the Pythagorean theorem)

   Special cases: (*MEMORIZE or be able to derive*)
   A. 45 degree triangle: both sides are the same, so the sin = opp / hyp = adj / hyp. From the Pythagorean theorem, opp**2 + adj**2 = hyp**2 = 2 * opp**2, so   sin (45 deg) = cos (45 deg) = sqrt (1/2) ~ 0.7071

   B. 30-60-90 triangle: the sin (30 deg) = 0.5 (exactly).   So for that case, opp = hyp / 2.
    So the adjacent arm is given by Pythagoras as opp**2 = hyp**2 - adj**2 = hyp**2 – (hyp**2)/4.
    Thus the cosine of 30° (= sin of 60°) = sqrt (3/4) = 0.866

   C. **small angles:** for small angles, the sine of the angle is approximately the same as the size of the angle IF MEASURED IN RADIANS (see below). So for a ten degree angle, the angle in radians is about 10/57.3 ~ 0.1745.   The sin of 10 degrees is actually 0.1736.   The smaller the angle, the more accurate this is.   So, for small angles sin (alpha) ~ alpha/57.3   (if alpha is measured in degrees). Often used for getting angular size of distant objects.

   Google calculator: If you don't have a trig calculator, you can type into google "what is sin(10 degrees)?" CAREFUL: If you ask "what is sin(10)?" you will get the sin of 10 RADIANS.   (try asking it a lot of things… "what is the mass of Mars"? kinda cool)

19. **angle measured in radians:** the arc length of an angle (the portion of the circumference that the angle subtends) divided by the radius (R). (for distant objects, the size over the distance). A full circle is 2 pi radians so one radian is about 57.3 degrees (360/ 2pi).

20. **Corotation speed** at locations NOT at the equator: The distance traveled in one spin (day) is the circumference of circle of the parallel of latitude. The radius of that circle is R cos (λ), where λ is the latitude. So the corotation speed is at a latitude λ is given by

21. **Volume of a sphere** $V = 4/3 * pi * r**3$ where r is the radius
    so the density ρ (rho) of a sphere = m / V where m is the mass

22. **Surface area of a sphere:** $4 * pi * r**2$ where r is the radius

23. **Drag force:** proportional to Area * v**2 (cross-sectional area times velocity squared). Increases with increasing speed, so at some point the drag force equals the gravity force. At that point no more acceleration occurs (since there is no more "net force"), so the falling object reaches "terminal velocity". For most objects on Earth this is 300 MPH or less, but it can be reduced by maximizing the "cross-sectional area" – the area perpendicular to the motion. That's why skydivers "spread eagle" themselves…. To slow down. If you are falling out of a building or an airplane, try it (and head for a nice bush). It might save your life.

24. **Precession**: movement of the Celestial Pole (and therefore also the vernal equinox) because of the changing direction of the Earth's spin axis, caused by tides raised by the Moon. Goes around once in 26,000 years. Star charts use "epochs" (1950, 2000, etc.) since BOTH coordinates change slightly. (and your zodiac sign should change, too!)

25. **Solar day (synodic day)**: 24 hours = 86,400 seconds. Noon to noon (sun aligned).

26. **Sidereal day**: 24 hours – 4 minutes. Star overhead to the same star overhead again, which is the TRUE rotation period of the Earth (or planet). For Earth, shorter than 24 hours because of the Earth's revolution around the Sun, about one degree per day.

27. **Logarithm**: if you write a number as a power of ten, the logarithm is that power, which can be a fractional number. Examples: $\log (1000) = \log (10**3) = 3$. $\text{Log} (\text{sqrt} (10)) = \log (10**.5) = .5$ And since sqrt (10) is approximately pi, then $\log (pi) = .5$ and $\log (\text{sqrt}(pi)) = 1/2 (\log(pi)) = 0.25$
    definition: $\log (10**x) \equiv x$

28. Multiplying numbers, you add the logarithms; dividing numbers, you subtract the logarithms. This is how a slide rule works, marked out as logarithms. *Know how to use a slide rule to multiply, divide, get logs, take square roots, etc.*
    $\text{Log} (a * b) = \log (a) + \log (b)$
    $\text{Log} (a/b) = \log (a) - \log (b)$

29. Logarithm of powers = multiply the logs
    $\text{Log} (a ** b) = b \log (a)$
    $\text{Log} (\text{bth root of } a) = (1/b) * (\log (a))$ *(be able to get fractional roots of numbers!)*

30. **log-log graph paper**: plots of the log of one number versus the log of the other. So, it is great for plotting relationships that are POWER LAWS of one another, like Kepler's third law:
    $T**2 = k * a**3$ so, taking the log
    $2 \log T = \log (k) + 3 \log(a)$
    $\log T = 1/2 \log k + 3/2 \log a$ (dividing by 2)
    so if y = log T and x = log a, you get a line with slope 3/2 and intercept (log k)/2
    so, for the log-log plot of satellite periods and distances, the intercept is (log k)/2

and for Kepler's law, the Kepler k is given by $(2 \text{pi})^{**}2 \, / \, G \, M$ where M is mass of the central body!

so the ratio of two planets's Masses can be calculated from the offsets of the two lines.

Intercept 1 = $\log (k1) \, / \, 2$

Intercept 2 = $\log (k2) \, / \, 2$

Intercept 1 – intercept 2 = $(1/2) \, [\, (\log \, ((2 \text{pi})^{**}2 \, / \, G \, M1) \, - (\log \, (\, (2 \text{pi})^{**}2 \, / \, G \, M2)]$
= $(1/2) \, \log (M2 \, / \, M1)$

So, you take the offset of the two lines (in the log, by seeing where the lower trace crosses an even power of ten and then taking the value of the upper), and squaring it, to get the value of the ratio of the masses.

31. **ellipses**:   an oval with two foci, such that the sum of the distances from any point on the curve to the two foci are a constant. Is a "conic section" – a cut across a "right" cone (a cone with a vertical central axis). If the cut is perpendicular to the axis, the ellipse is a circle. If at an angle to the axis, the ellipse is elongated (eccentric).

32. **major axis**: long measurement of an ellipse (along line with the foci). Length = 2a
(a is defined as the **semi-major axis**).

33. **minor axis**: shorter distance of an ellipse measured perpendicular to the major axis.
Length = 2b (b is defined as the **semi-minor axis**).

34. **eccentricity**: ratio of the distance between the two foci of an ellipse to the major axis
$e \equiv$ (distance between foci) / 2a

35. **apogee, aphelion, apoapsis**: longest distance from a point on an ellipse to the focus, for objects orbiting the Earth, the Sun, or a general object, respectively.
$R_{ap} = a \, (1+e)$

36. **perigee, perihelion, periapsis**: shortest distance from a point on an ellipse to the focus, for objects orbiting the Earth, the Sun, or a general object, respectively.
$R_{pe} = a \, (1-e)$
You should be able to calculate a and e from R_{ap} and R_{pe} or vice-versa (know any two, get the other two). Distance between foci = ae $= (R_{ap} – R_{pe})$. Major axis= 2a $= (R_{ap} + R_{pe})$.   So,
$e = (R_{ap} – R_{pe})/(R_{ap} + R_{pe})$ (add or subtract formulas 34 and 35 to get these).

37. **centripetal acceleration**: the acceleration needed to make something turn in a circle. For an object moving at speed v in a circle of radius r, the centripetal acceleration is negative (towards the center of the circle). The centripetal force is the necessary force = m * **a**.
$\mathbf{F}_c = - m \, v^*v \, / \, \mathbf{r}$

38. **centrifugal force**: the fictitious force equivalent to the opposite of the centripetal force. Objects in a moving car feel the "force" outward when it accelerates around a curve. Not a true force.. just felt in a non-inertial frame of reference.

39. **gravitational force**: the force of gravity of two bodies, one with mass M and one with mass m, on each other. The force "falls off" with distance as 1/(distance squared). The sign of the force is negative because it is attractive.
$\mathbf{F}_G = - (G \, M \, m) \, / \, \mathbf{r}^{**}2$
where G is the gravitational constant = 6.67 E-11 (N-m**2/kg**2).
So if you put in the mass in kg and the distance in m, it will give the force in Newtons (N).

note: since F = ma, one Newton is also one (kg * m) / s**2

40. **escape velocity**: the speed at which, if a particle is leaving the surface (and no atmospheric drag), it will end up at infinity, but at rest.
    An easier way to think of it is, if you drop an object from infinity, and it freely accelerates to the surface (no drag), the speed at which it hits the surface is the escape velocity.
    The escape velocity includes the effect that the acceleration of gravity gets weaker and weaker the higher you are.
    Formula: $(1/2) m * v_e^2 = G * M_p * m / R_p$
         (the kinetic energy at the surface = the potential energy at infinity)
    Where R_p is the radius of the planet, v_e is the escape velocity from that planet, and M_p is the Mass of the planet.   Notice the mass of the object drops out!
    So $v_e = $ sqrt (2 $G * M_p / R_p$)

41. **orbital velocity**: the speed in orbit of an object. From Kepler's and Newton's laws, it does NOT depend on the mass of the orbiting body, just the mass of the CENTRAL body, so long as the orbiting body is a lot smaller than the central body.
    For a body of mass m in circular orbit at a distance from the center of the planet R, the centripetal acceleration = the gravitational acceleration

    m v * v / R   = G M_p m / (R*R)

    so v = sqrt ($G * M_p / R$)

    note how similar this is to the escape velocity! For an orbit near the planet, R is just over R_p, so
    v (orbital) = v (escape) / sqrt (2)

42. Plugging in values for Earth, v_o (low earth orbit) is around 8 km/s and the escape velocity v_E is about 11 km/s.

43. From this and Kepler's laws, you should be able to calculate the speed of the Moon in its orbit around earth.

44. **geosynchronous orbit** = the orbit at which the orbital period = 1 day
    This orbit is very useful for communication satellites.
    You should be able to calculate that distance from the center of the Earth is about 6.6 R_E.

45. Derivation of Kepler's third law: From the orbital velocity calculation (number 40) you should be able to derive Kepler's 3rd law ($T^2 = K a^3$).   What does K turn out to be for an object in orbit around a planet with mass M_p?
    Be able to, if given a satellite a and T, then calculate the central mass. Or if given the mass, and a, calculate T, etc.

46. **Be able to:** express a number in **scientific notation**
    use **significant figures**: 1.68×10^{14} has 3 significant figures; 1.680×10^{14} has 4 significant figures;   16804000. has 5 significant figures.   *Don't give me more figures back in your calculation than I gave you in the problem. (well, maybe one more, but not 5).*
    change from decimal degrees to degrees:minutes:seconds
    know the **phases of the moon** relate to the time of day seen (full moon rises at sunset, etc.)
    figure out the **sidereal time** of special dates (equinox and solstice) at noon, midnight, etc.
    calculate the **perihelion** and **aphelion** distances given a and e, or vice versa

calculate the **density of a sphere**
calculate the **escape velocity** or **orbital velocity** of a planet given its mass and radius
calculate the **surface gravity** of a planet given its mass and radius
know the log of 2 (.3) and the log of pi (.5) – from those you can estimate the rest…

47.   Know that the Moon and Sun are each about a half-degree in the sky. Know that the Moon's diameter is about 1/4 of Earth's. Know that the Moon's distance from the Earth is about 9.5 Earth circumferences (3.84×10^5 km = 60 R_E).

48. The Sun. Know that the Sun is mostly hydrogen with about 10% Helium and small amounts of heavy elements. Extra heavy elements means that our sun is a second generation star – a previous supernova exploded and created heavy elements that our Sun absorbed. That it is powered by nuclear fusion in the core. Energy is transferred by conduction (core), radiation (outer core), convection (upper levels), and radiation again (photosphere). Know photosphere, chromosphere, corona, filaments, prominences, solar flares, coronal mass ejections. Know that we know the composition of the solar atmosphere by absorption lines, the composition of the corona from emission lines, and the composition of the solar wind from sheets of foil from Apollo (Long duration exposure facility) and from the glass plates of the Genesis mission (which crashed but still got some useable data).

49. Wien's law: light from a black body has a continuous spectrum with peak of spectrum at
    $\lambda = 0.3$ (cm) / T(K). Know how to change wavelength to speed and frequency
    $\lambda = c/f$ where λ is wavelength (m/osc), c is speed of light (m/s), and f is frequency (osc/s =Hz)
        = 300 MHz / f

Constants, etc…

c = speed of light = 3×10^8 km/s = 3.0×10^5 km/s
1 AU = distance from Sun to Earth = 1.5×10^8 km = 1.5 E11m
seconds in a year = $365.25 \times 86400 = 3.15 \times 10^7 \approx \pi \times 10^7$
1 ly = distance light travels in a year = $3 \times 10^5 * 3.15 \times 10^7 = 9.5 \times 10^{12}$ km
R_E = radius of Earth = 6378 km
R_M = radius of the Moon = 1737 km
R_{SUN} = radius of the Sun = 6.955×10^5 km
M_* = 1.99 E30 kg (Sun) and M_E = 5.97 10^{24} kg (Earth)
G = $6.67 * 10^{-11}$ (N m^2 /kg^2)

Log table
Log (1) = 0.   Log (2) = .30   Log (3) = .477
Log (pi) = .5   Log (4) = .60   Log (5) = .699
Log (6) = .778
Log (7) = .85   (789 rule: log (x) = .85 + (x-7)/20) for x between 7 and 10)
Log (8) = .90
Log (9) = .95
Log (10) = 1.0
Log (20) = 1.3
Log (100) = 2
Log (200) = 2.3   etc…