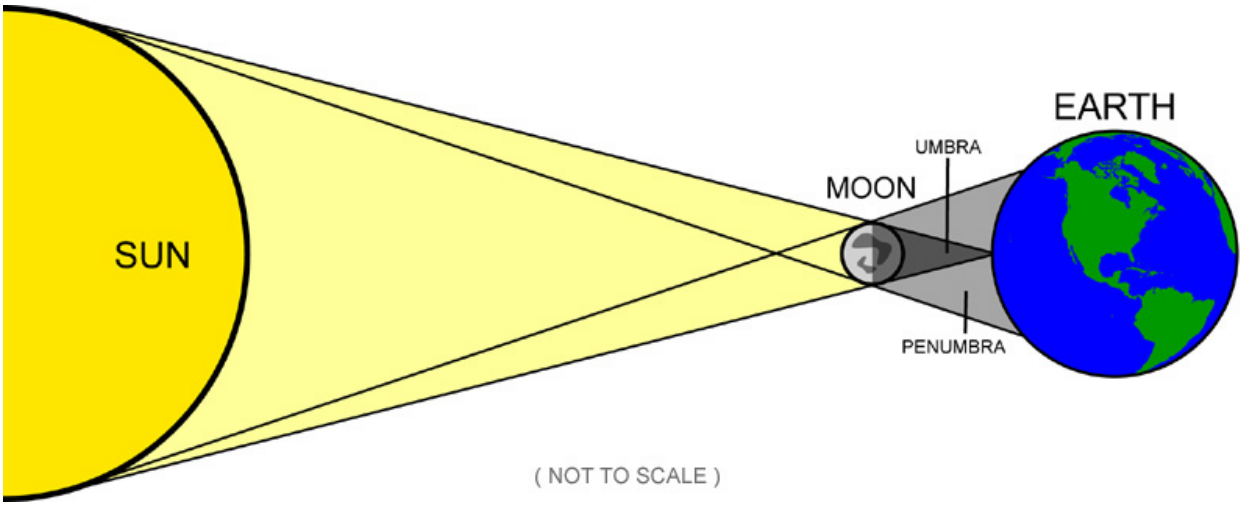
Eclipse Math - Proportions and Angular Size

Patricia Reiff, Rice University



The Moon can appear to cover the Sun because the Moon, even though 400 times smaller than the Sun, is 400 times closer than the Sun!

The angular size of an object, in degrees, is given by

Angle (deg) = 57.29 (Length/Distance)

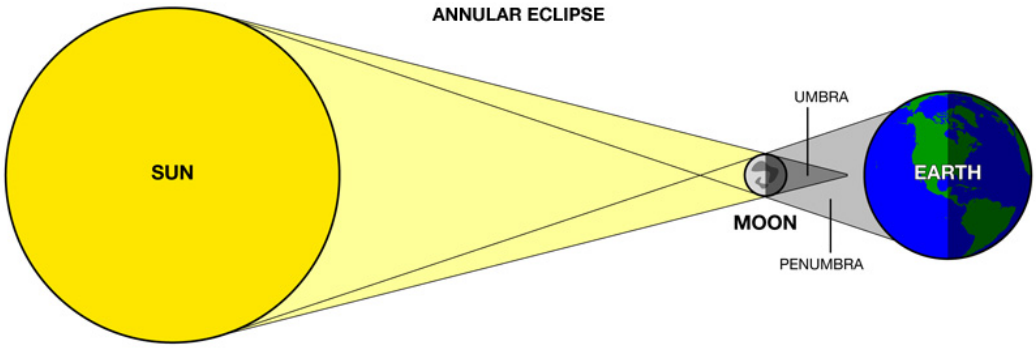
So, if the Sun is 1.495×10^8 km away, and its diameter is 1.391×10^6 km, what is its angular size in degrees? (for younger children use 1.4 million km and 150 million km)

Let's do that again, now let's look at the Moon.
If the Moon's diameter is 3474 km and its distance is 384,400 km, what is its angular size in degrees? Actually, however, we want the distance from the Earth's ***surface***, not from the center, so we have to subtract the radius of the Earth = 6378 km, so that makes the distance = 378,000 km. For the Sun it doesn't matter much, but for the Moon it does.

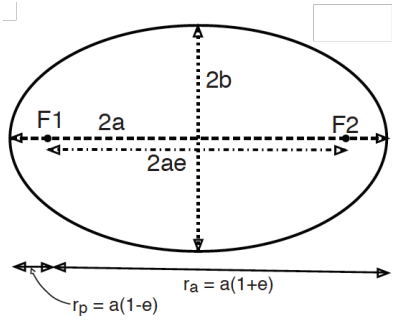
So, these numbers are about the same, but which number is bigger? So which object is a tiny bit larger in the sky in angular size?

So, can the Moon appear to completely cover the Sun? What is the problem here?
We know total eclipses happen - what is the solution?

In fact, ***on average***, the Moon doesn't quite cover the Sun… it leaves a little bit of the sun uncovered. This is an **annular** eclipse.



The solution is that the distance from the Earth to the Moon, and from the Earth to Sun, is not a fixed number. The Earth's orbit makes an **ellipse** around the Sun and the Moon's orbit is an **ellipse** around the Earth. The amount of ellipticity of an orbit is called the eccentricity (e) and is the fraction that the maximum distance, apogee, (r_a) **exceeds** the average distance (a). It's also the fraction that the minimum distance at perigee (r_p) is **less** than the average distance. We were using the AVERAGE distance before, and on average, the Moon does NOT completely cover the Sun!



r_a = (1 + e) a **apogee or aphelion = maximum distance**

r_p = (1 - e) a **perigee or perihelion = minimum distance.**

So, if the eccentricity of the Earth's orbit around the sun is 0.0167, what is the **maximum** distance of the sun away from the Earth? (to make the Sun's angular size **smaller**)

Using that distance, what is the minimum angular size of the sun?

And, if the eccentricity of the Moon's orbit around the Earth is 0.0554, what is the **minimum** distance of the Moon away from the Earth? (to make the Moon's angular size **larger**)

Using that distance, what is the maximum angular size of the Moon? (Again, subtract 6378 km for the distance to the surface)

Extension: *The moon is moving away from us at 4 cm / year. How long will Earth still enjoy total eclipses?*